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Augustus De Morgan and the propagation of moral mathematics

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Abstract

In the early nineteenth century, Henry Brougham endeavored to improve the moral character of England through the publication of educational texts. Soon after, Brougham helped form the Society for the Diffusion of Useful Knowledge to carry his plan of moral improvement to the people. Despite its goal of improving the nation's moral character, the Society refused to publish any treatises on explicitly moral or religious topics. Brougham instead turned to a mathematician, Augustus De Morgan, to promote mathematics as a rational subject that could provide the link between the secular and religious worlds. Using specific examples gleaned from the treatises of the Society, this article explores both how mathematics was intended to promote the development of reason and morality and how mathematical content was shaped to fit this particular view of the usefulness of mathematics. In the course of these treatises De Morgan proposed a fundamentally new pedagogical approach, one which focused on the student and the role mathematics could play in moral education.

Keywords: Mathematics; Education; Henry Brougham; Augustus De Morgan; Nineteenth-century England.

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1. Introduction

Lord Brougham circulated a treatise in March 1827 promoting the relevance and benefits of a scientific education in England. Although he enunciated many practical reasons for learning mathematics and natural philosophy, for him the greatest pleasure of their study was religious:

> The highest of all our gratifications in the contemplations of science remains: we are raised by them to an understanding of the infinite wisdom and goodness which the Creator has displayed in his works... The delight is inexpressible of being able to follow, as it were, with our eyes, the marvelous works of the Great Architect of Nature—to trace the unbounded power and exquisite skill which are exhibited in the most minute, as well as the mightiest parts of his system.[1]

Science enabled its practitioners to see beyond the mundane to the wonderful. Significantly, Brougham was not a scientist. He was a career politician who would go on to hold one of the highest offices in Great Britain, Lord High Chancellor. In the 1820s, however, his writings and projects grew out of the vibrant intersection of academics, religion, and politics.

The connection Brougham saw between science and religion was the hinge upon which much of his personal moral philosophy turned. He believed that the state of morals in England was reprehensible, especially among the lower classes, and believing the solution to be in the promotion of self-education, he founded the Society for the Diffusion of Useful Knowledge (SDUK). He intended the Society to promote morality through secular education, especially in the sciences. For Brougham, subjects such as mathematics were not abstract and inapplicable, but intimately connected with a person's daily moral and religious experiences. Brougham and his Society needed a mathematician, however, to propagate this form of 'useful' knowledge to the nation. For that, he turned to a recent graduate of Cambridge University, Augustus De Morgan. Through De Morgan, Brougham was able to carry his plan of moral improvement to the people with the Society for the Diffusion of Useful Knowledge as the medium through which such dissemination occurred.

This article will trace the ways in which De Morgan shaped mathematics to his particular conception of logic and rigor and the resulting means by which his texts served the moral purpose of the SDUK. Historians have already begun to uncover the dominant forces within the educational, moral, and scientific spheres in this period, especially in the work of Adrian Rice and Andrew Warwick.[2] Rice has traced De Morgan's pedagogical work within the University of London, and Warwick has examined how the training of men transformed the study of physics at Cambridge University. The latter's book, *Masters of theory*, brings to the forefront

[1] Brougham (1827), pp. 181–182.
[2] Rice (1997), Warwick (2003); see also Warwick (1998) and Rice (1996).

how alterations in the material culture of examinations and the tutorial system of education had profound implications for scientific pedagogy and the production and reception of new knowledge.

Nonetheless, little attention has been paid to the actual content of published texts, even as the mathematical treatise was of increasing pedagogical importance in the early nineteenth century. Neither Rice nor Warwick was concerned with the way that mathematical knowledge was propagated to the masses; both concentrated on the production and acceptance of knowledge within the university setting. At that time, affordable treatises were perceived as a means of assisting the untutored student and broadening the influence of mathematics. As the primary mathematical author for the SDUK, De Morgan's treatises mark an important shift in the manner in which mathematics was conceived and taught. This shift, when considered concomitantly with the changing place of mathematics within the university, exposes a fascinating moment within the history of mathematics. Pedagogical concerns were shaping mathematics itself. De Morgan's treatises revealed how his own mathematical assertions were inextricably linked to the social and moral concerns of the Society. This episode is one case where studying the history of mathematics—seemingly an insulated field—is impossible without considering how and why the subject was being propagated politically to the masses.

In the sections below, I explore the new kind of mathematics De Morgan was espousing in his writing for the SDUK. In self-consciously rejecting previous models of how mathematics ought to be learned, De Morgan aimed to displace the old canon of mathematical pedagogy through his work for the SDUK. My analysis will focus on how these new techniques were an integral part of the moral mission of the SDUK. De Morgan was reshaping mathematics in the context of Brougham's moral Society, and through examining De Morgan's philosophy, we will see how mathematics had been reshaped as a logical and moral activity. Finally, by widening the scope and introducing the philosophical and religious texts of William Whewell and Brougham, we can contextualize the treatises in the ongoing discussion about the relationship between religion and education.

2. The society for the diffusion of useful knowledge

Although it is not my intention to provide a history of the SDUK, we need to understand its mission as the context for De Morgan's pedagogical texts. The first meeting of the SDUK was held in London on 6 November 1826. The men were gathered—according to the minutes—to address the need for moral improvement of the population:

> Mr. Brougham explained the purport of the meeting, and pointed out the importance and necessity of some Plan of Co-operation for the moral

improvement of the great body of the labouring Population, & the Means by which this Object may be most effectively attained.[3]

The adjective 'labouring' was crossed out in the original minutes of the meeting. Although the author of the agenda initially intended the SDUK to be focused on the working class population, the general committee apparently decided to have a broader mission. Critics quickly arose, however, who claimed that poor laborers did not need more than rudimentary education and only the upper classes could afford such leisure time. Brougham responded that the Society aimed to diffuse knowledge broadly, though he realized the upper and middle classes would initially benefit more. He believed the SDUK's publications 'are adapted to all ranks of the community', but was 'well aware that improvement always begins at the higher, and descends from thence to the humbler classes'.[4] Brougham hoped to improve all levels of the population, but realized he could only effectively create the desire for education through the upper and middle classes.

Although the stated goal of the SDUK was to kindle the desire for education for the purpose of improving the morals of the country, the vast majority of its publications were not explicitly moral or religious. As Brougham's initial treatise on the benefits of science suggests, many of the publications presented topics in natural philosophy and mathematics. Yet treatises on mathematics seemed abstruse and abstract—certainly not something the working classes would have found to be 'useful knowledge'. Some arithmetic may have been practical for laborers, but treatises on the differential and integral calculus and on algebraic equations presented the working class with material that was both difficult to master and rarely applicable in their lives. It was not, however, for application that such material was deemed useful. The inclusion of substantial mathematics in the output of an organization devoted to producing useful knowledge followed from the conviction that mathematics could play a central role in moral improvement. The absence of an emphasis on mathematics' practical applications should not obscure the important ways the nature of the material itself was found to be useful. Brougham's own conviction of the connection between religion and mathematics will be central to resolving this apparent chasm between useful knowledge and abstract mathematics.

Brougham's program of mathematical education was largely carried out by Augustus De Morgan, who stands today as one of the most important figures in English mathematical history and pedagogy of the nineteenth century. De Morgan was the principle author of mathematical texts for the SDUK and also one of the leading figures in mathematical education generally in this era. Born in India to English parents in 1806, De Morgan graduated from Trinity College, Cambridge, in 1827.[5] Before the end of his graduation year, he had already been commissioned

[3] General Committee Minutes (GCM), 11/6/1826, SDUK Archive, UCL.

[4] Brougham (1829), p. 190. Although the philosophical and political implications of Brougham's strategy are fascinating, they are beyond the scope of this investigation.

[5] The only comprehensive biography of De Morgan was written by his wife soon after his death: S. De Morgan (1882).

to write a treatise on statics for the SDUK, and word of De Morgan's superb mathematical ability spread quickly. Brougham's involvement with the founding of both the University of London and the SDUK brought him into regular contact with De Morgan during the late 1820s and set the stage for their continued collaboration on the SDUK's moral mission.

3. Mathematical education in nineteenth-century England

Mathematics was already being taught in England in the early nineteenth century and was playing an ever more crucial role in the country's understanding of a liberal education. De Morgan's contribution was a new conception of how mathematics ought to be taught, a pedagogy shaped with the principles of the SDUK in mind. By understanding how De Morgan displayed his own pedagogical beliefs in his writings, we can see both their novelty and significance and the ways De Morgan's beliefs justified some of his editorial choices in the SDUK treatises.

The most substantial repositories of De Morgan's educational work are the SDUK's *Quarterly Journal of Education* and a treatise he produced for the Society's 'Library of Useful Knowledge', *On the study and difficulties of mathematics*. De Morgan was one of the primary authors for the journal, which from 1831 to 1835 sought to 'diffuse a fair and unbiassed [sic] criticism on establishments for education, and on the systems and the books which constitute their real life and existence'.[6] His *Study and difficulties of mathematics* was the central didactic treatise for the Society, providing his analysis of how mathematics ought to be learned in the context of actual mathematical instruction. These publications expose a pedagogy fundamentally different from contemporary theories. De Morgan's treatises and articles assert that even in cases of difficult or abstruse topics, students must appeal to reason and logic, thereby transforming the subject to one which is essentially rational.

In the early nineteenth century, mathematics had little place in the standard curriculum beyond the arithmetic needed for daily transactions. The very structure of schooling available to rich and poor children alike inhibited meaningful mathematical study. The widespread use of 'monitors' in schools saved money but also made elder students the only instructors of their younger peers. This resulted in the use of rote memorization taught by those who understood little, to those who understood less. Mathematics beyond the level of numerical manipulation was impossible in this setting.[7]

Fighting for an increased presence of mathematics were De Morgan and William Whewell, working at the institutions of the University of London and Cambridge University, respectively. De Morgan was hired as the first Professor of Mathematics at the University of London in February of 1828, a post he held until 1867, except for his brief resignation in the 1830s. De Morgan's work paralleled the continuing

[6] Society for the Diffusion of Useful Knowledge (1831), p. 2.
[7] Howson (1982), p. 102.

work of Whewell—De Morgan's undergraduate instructor—at Cambridge. Whewell is largely credited with stating the case for Cambridge's emphasis on mathematics in his volume, *Thoughts on the study of mathematics, as part of a liberal curriculum*.[8] Whewell believed mathematics was a way to learn to reason and that the subject, when combined with a religious base, would promote the development of gentlemen prepared for all walks of life. De Morgan generally agreed that mathematics ought to play a significant role in university education, but asserted that mathematical education should begin much earlier in a person's life.

Whether this mathematical focus would be able to trickle down from the university elite to the lower classes depended on two factors: the existence of mathematically sound instructors and the availability of useful mathematical texts. The first factor has already been largely dismissed: even by the 1850s, primary schools did not include mathematics in their curriculum in any serious way. The availability of texts also posed a problem. Warwick has examined the problematic rise of print culture within mathematical education. Using his analysis as a basis, we can see the way that De Morgan and the SDUK's publications fit within the changing culture of printed texts.

Warwick has noted that referring to 'mathematical textbooks' is deceptive, as no such thing existed in the nineteenth century as we currently conceive them. Texts were neither comprehensive nor pedagogically concerned, and thus were aimed as much at the professional mathematician as at the beginning student.[9] This complemented the Cambridge system of tutor-based study where students had private coaches lead them through the mathematical developments and prepare them for the problems on the Tripos. However, most students across the country were left without any means of self-study. Further inconveniencing the novice, many of these texts were of extraordinary difficulty, with problems geared increasingly toward the Tripos student. As the ability to solve problems on paper became more crucial to the Cambridge system of collegiate honors, the appearance of preparatory texts became more widespread. This initiated a cycle in which examiners increased the difficulty of questions, and texts correspondingly increased in sophistication to account for the prevalence of preparatory books. In fact, fluxional calculus textbooks were often more complicated in the mid-nineteenth century than they were at the turn of the nineteenth.[10] De Morgan had a particular grievance with this increasing focus on Tripos examinations: the reliance on problem sets distracted from the clarity of exposition De Morgan believed was central to mathematical education.

De Morgan, whose own Tripos experience labeled him fourth in the university—honorable, but below his tutor's expectations—consistently denounced an intensive textual focus on problems throughout his life. Toward the end of his teaching days, in 1865, he gave the inaugural address as the first president of the London Mathe-

[8] Whewell (1836).

[9] Warwick (2003), pp. 141–142.

[10] See ibid., p. 142 ff., who cites Guicciardini (1989), pp. 125–126.

matical Society in which he outlined the kind of mathematics based on problems he hoped to see avoided in the coming decades:

> Now the English mathematical world of the present day takes its tone principally from the Cambridge examinations; there is no doubt of that, and there is no use in denying it. The Cambridge examination is nothing but a hard trial of what we must call problems—since they call them so—between the Senior Wrangler that is to be of this present January, and the Senior Wrangler of some three of four years ago. The whole object seems to be to produce problems, or, as I should prefer to call them, hard ten minute conundrums. These problems, as they are called, are, and are necessarily obliged to be, things of ten minutes or a quarter of an hour. It is impossible in such an examination to propose a matter that would take a competent mathematician two or three hours to solve, and for the consideration of which it would be necessary for him to draw his materials from different quarters, and see how he can put together his previous knowledge, so as to bring it to bear most effectually on this particular subject.[11]

De Morgan attempted to rectify these deficiencies in texts in his own publications. None of his texts, nor any of the SDUK's, included explicit problems for the student to complete. The books were for students, and the absence of problems meant that teachers could not use them as examination materials.

De Morgan's greatest fear about problem based texts was that they led students to a blind acceptance of particular absurdities. Without a knowledgeable teacher to watch over each step, mistakes memorized from texts could propagate freely, creating contradictions disguised as legitimate mathematics.

> If used without proper caution, they [certain textbooks] lead the student into erroneous notions, which some elementary works, far from destroying, confirm, and even render necessary, by adopting these very notions as definitions; as for example, when they say that a negative quantity is one which is less than nothing; as if there could be such a thing, the usual meaning of the word less being considered, as if the student has an idea of a quantity less than nothing already in his mind, to which it was only necessary to give a name.[12]

This potential hazard provided De Morgan's impetus for writing texts. If good teachers were unavailable and the books were geared toward mathematicians and tutored students, De Morgan needed to write for the extensive group with neither quality texts nor careful instruction.

De Morgan expected his treatises to augment the literature in two important ways. First, he hoped that they would assist the otherwise unaided student. Second, he ensured his textbooks were focused on the student's pedagogical desires. His primary objective, as stated at the start of *Study and difficulties* was 'to point out to the student of Mathematics, who has not the advantage of a tutor, the course of study

[11] De Morgan (1865), pp. 1–2.
[12] De Morgan (1830), p. 49.

which it is most advisable that he should follow, the extent to which he should pursue one part of the science before he commences another, and to direct him as to the sort of applications which he should make'.[13] De Morgan aimed to give students a plan of study, the focus of which would not be the amount a student could acquire but the proper order of learning and the proper end for the student—the inculcation of reason.

De Morgan saw the order of study of mathematical subjects as an important consideration in the publication of treatises. In *Study and difficulties*, his primary SDUK treatise on the trials students face in the subject, he begins with a discussion of the difficulties inherent to arithmetic and basic numbers, especially with negatives and impossible quantities. Then, he proceeds to show how these difficulties were handled in algebra, finally describing the geometric subtleties of proportion, incommensurables, and the necessary precision of definition. The order of his publications for the SDUK reveals that he intended the use of supplemental treatises on applications of the calculus and examples of the processes of arithmetic and algebra. De Morgan made explicit the didactic reasons for the existence of such supplemental treatises in a letter to the SDUK's secretary, Thomas Coates, about De Morgan's upcoming book on the *Differential and integral calculus*:

> As I told you some time ago, the Treatise on the Differential Calculus[,] the first part of which is now in your hands, will be very hard for the generality of beginners, as well any other written on the common place... The reason is that new principles and those [that] are not very easy are not exemplified by sufficiently simple mathematical examples, and moreover the student is required to learn a new and different notation, and to apply new rules to every sort of mathematical expression, before he can have any conception of the ultimate use of what he is about.
>
> To try to do some good in this line instead of emitting at once the Treatise I have sent you, I propose to write an introduction, which may in one sense be called a separate work, if your Committee think the plan may be useful. It would be called 'Elementary Illustrations of the Differential and Integral Calculus,' and would be two numbers at most or which would be more desirable one only if the requisite matter could be got in without making clearness yield to concreteness. It would not contain the explanation or the general Precepts of the Diff. Calc. But would take certain easy examples of curves forces and other things which are handled by the method of limits.[14]

In cases where students would have difficulty with the initial postulates or notation, De Morgan felt it was worthwhile to motivate such assumptions with examples. He claimed that by giving examples he could make such postulates acceptable without sacrificing the clarity of propositions. Once they were established, De Morgan could

[13] Ibid., p. 1.
[14] In letters, De Morgan to Coates (3/26/1832), SDUK Archive, UCL.

present the more general precepts of the calculus, sure that as many people as possible would benefit. This kind of thoughtfulness about *how* to study characterized the majority of his publications.

In addition to assisting the student, De Morgan's second requirement was that his textbooks be focused on the student. He believed, as did Whewell, that students most benefited when an author included comprehensive and up-to-date information.[15] To counter the prevalence of textbooks focused on only one particular methodology, he advocated giving students access to the best techniques without requiring them to read every published text. De Morgan's calculus text, for instance, begins with a discussion of the different historical ways of doing problems, and his articles in the *Penny Cyclopedia* on analytic notation similarly focus on the different potential solutions. In *Differential and integral calculus*, he takes so seriously the imperative to present all useful options that he includes the method of infinitesimals bracketed within his text even though he explicitly labels it as an erroneous method. The method was included simply because it might help students' conceptual understanding. Whereas other textbooks focused on the mathematics, De Morgan focused on the student, and shaped the mathematics to this particular conception.

A telling example of these different foci arises from a comparison between one of the most common textbooks of the time, John Bonnycastle's *Algebra*, and De Morgan's work for the SDUK. De Morgan cites Bonnycastle's book as one example of a text students learn by memorization.[16] The text itself is geared toward this kind of memorization. Bonnycastle begins by giving the major definitions used in algebra. He avoids all tricky situations and concepts, such as negatives and imaginary numbers, instead focusing on short descriptions of simple arithmetic and algebraic properties that can be easily memorized. Then, each topic is divided into different cases: one case of addition, for instance, is when the signs of the quantities to be added are the same. By memorizing the rules to carry out each case, a student would only have to recognize which case a given problem fell under and apply the appropriate rule. Having memorized all cases and rules, algebra simply became a game of manipulation based on matching problem to rule.

A more extensive example elucidates the style of Bonnycastle's text. Case VI of the section on algebraic fractions, for example, is 'to add fractional quantities together'. The rule is deceptively simple: 'Reduce the fractions, if necessary, to a common denominator; then add all the numerators together, and under their sum put the common denominator, and it will give the sum of the fractions required'.[17] This is neither a conceptual treatment where fractions are considered as part of a pie, nor a rigorous mathematical discussion where reasons for supplying a common denominator are given. There are no applications to real objects at any point in the textbook; there are only rules of manipulation without regard to meaning. This kind of analysis was the source of De Morgan's frustration with the state of mathematical education. In De Morgan's texts he justified his actions, gave demonstrative

[15] Warwick (2003), p. 147.
[16] De Morgan (1831), p. 266, and Bonnycastle (1825).
[17] Bonnycastle (1825), p. 37.

examples, and provided the student with the information needed to understand the different steps involved in the solution of the problem.

In the work of the Society, De Morgan's new methods took precedence over that of Bonnycastle, revealing the SDUK's attempts to shift the focus of mathematical pedagogy. The difference between the two authors is similar to that between inductive and deductive reasoning. Whereas De Morgan wanted the focus to be on the process, inducing the greater laws governing the operation, Bonnycastle believed students learn best when given the rules to memorize and then left to deduce the proper application of them based on the given problem. This triumph of 'inductive learning' is echoed in an article by Brougham about the SDUK.

> We discover new truths, no doubt, by proceeding from things known, to things unknown... When we would communicate the knowledge thus gained, however, it is said, we reverse the process—begin by stating the general rule, and then show how the instances range themselves under it. We cannot help thinking that this view of the subject is ill founded, and that the analytical form is better fitted for didactic purposes, generally speaking, than the synthetic.[18]

De Morgan's rejection of Bonnycastle's system was consistent with Brougham's general views on scientific education, and placed the emphasis on the ability of the student to comprehend the problem or situation.

4. De Morgan's new pedagogical philosophy

In returning the focus to the process of learning, De Morgan established a new pedagogical basis for mathematics, one that would ultimately fulfill the SDUK's moral mission. Although he never published a treatise as comprehensive as Whewell's on the role of mathematics in education, De Morgan anonymously published an article, 'On mathematical instruction', which outlined his beliefs about the subject.[19] The dominant theme was that mathematical education should focus more on inculcating reason than on applying mathematics to any particular pursuit. This conception required his fleshing out two points: first, there was little direct application for mathematics; second, mathematics must proceed from clear observations to logical conclusions.

4.1. Lack of application but abundance of utility

De Morgan's insistence that learning mathematics was useful primarily as a way to develop reasoning skills meant that the rote learning of arithmetic was useless. He granted that most monitorial schools ensured that the younger students were mem-

[18] Brougham (1829), p. 185.
[19] De Morgan (1831), pp. 246–278.

orizing the correct mechanisms for manipulating numbers, but criticized them for not focusing enough on why those mechanisms existed. Because most students could not spend the time necessary to study all the applications of the subject—especially if they belonged to the working classes—De Morgan suggested they ought to 'secure a habit of reasoning in preference to the knowledge of a host of results'.[20] Even if they could spend time learning applications, 'much of what is done has no reference to any practical end whatever'.[21]

Further, De Morgan thought measuring proficiency by the amount of Euclid students could recite missed the point of learning mathematics at all. For him, 'The actual quantity of mathematics acquired by the generality of individuals is therefore of little importance, when compared with the manner in which it has been studied, at least as far as the great end, the improvement of the reasoning powers, is concerned'.[22] This is not to say that De Morgan did not see applications for mathematics but rather that restricting the subject to its applications was overly limiting: 'In fact, there is not in this country any disposition to undervalue [mathematical skills] as regards the utility of their applications. But though they are now generally considered as a part, and a necessary one, of a liberal education, the views which are still taken of them as a part of education by a large proportion of the community are still very confined'.[23] There was little useful about memorizing formulae for algebra or arithmetic in De Morgan's view. The student's focus should instead be on the reasonable development of the proof.

To this end, De Morgan asserted that it was better to do mathematics than memorize its rules. Mathematics for him was not some particular body of knowledge of which a certain proportion ought to be memorized each month; the useful knowledge—the diffusion of which served as the aim of the SDUK—came only from understanding, not from solving, problems.

> We do not wish to depreciate its utility as an exercise for the mind, or to hinder all from attempting to conquer the difficulties which present themselves; but to remind every one that, if he can read and understand all that is set before him, the essential benefit derived from mathematical studies will be gained, even though he should never make one step for himself in the solution of any problem.[24]

In De Morgan's view, solving set problems was a benefit of, but certainly never the justification for, learning mathematics.

De Morgan blamed the overemphasis on problem solving on teachers who did not understand the underlying principles themselves. He noted that they resorted to rote examinations, graded by solution keys, to determine the quantity of knowledge in

[20] Ibid., p. 266.
[21] Ibid., p. 271.
[22] Ibid., p. 266.
[23] De Morgan (1830), p. 4.
[24] Ibid., p. 31.

pupils. De Morgan disliked advanced examinations such as the Tripos, so predictably, he also disliked a focus on problem solving instead of on principles and processes in the education of young children. He encouraged teachers to set up a proof system, which forced students to list every step singularly, with proper justification and explanation of the transitions and assumed theorems. Once teachers had done this, and put the focus on the process and not the conclusion, they could be assured that the student understood the problem.[25] It is not surprising to find this focus on the proof, as De Morgan's own mathematical development began with a conception of proof before any sustained investigation of facts or rules.[26] The proof, the steps, and the reasoning were vital for De Morgan.

4.2. Clear premises to logical conclusions

Once De Morgan established the focus on the process instead of the product, his second major requirement was that all mathematics should come from clear principles and follow to logical conclusions. He thus differentiated between the initial propositions and the development. 'Though the *certainty of mathematical conclusions* depends upon that of the fundamental propositions, the *correctness of mathematical reasoning* has nothing whatever to do with that circumstance'.[27] As far as his pedagogy was concerned, this differentiation had a number of implications. Most basic was that all mathematics should begin with clear premises. Without these, students never truly understand or critically examine the results that follow:

> From his earliest infancy, he learns no fact from his own observation, he deduces no truth by the exercise of his own reason... Thus a habit of examination is not formed, and the student comes to the science of algebra fully prepared to believe in the truth of any rule which is set before him, without other authority than the fact of finding in the book to which he is recommended.[28]

Good education and sound mathematics do not lead to blind acceptance for De Morgan, but to healthy skepticism. 'We know well that pupils always receive implicitly what their masters tell them, and why is it that they are led to the study of Mathematics? Precisely that they may learn to raise objections, and how to raise them in the proper place, when false logic and absurd definitions make objections desirable'.[29] In order to be able to question anything, however, children must begin to learn the correct fundamentals of the subject.

When first learning numbers, De Morgan asserted that children should learn to connect the name with the proper object. De Morgan complained that learning to

[25] De Morgan (1831), p. 276.
[26] Rice (1997), p. 18.
[27] De Morgan (1835), p. 95; emphasis in original.
[28] De Morgan (1830), p. 62.
[29] De Morgan (1831), p. 271.

'count' by the memorization of 'one, two, three, etc.' has no more to do with the matter of arithmetic than counting with the monikers, 'chair, sofa, table, etc.'[30] The name is inappropriately separated from the object and a child who is asked to give the meaning of 'three', is just as likely as not to hold up solely the finger which has been third in his reckoning. De Morgan asserted that this use of number leads the learner to imagine that 'three' signifies a tangible actuality, instead of the abstract quantity. The remedy for him was simple:

> Let a number of objects of several sorts be procured, say counters, marbles, and beans, and let the numerals never be used except in connexion [sic] with one of these; thus the child should never be allowed to say the word 'five,' except as part of one of the phrases, 'five marbles,' 'five counters,' or 'five beans'. Let the different collections of each of these be ranged in rows, from one up to five, and let the child proceed through each set separately, beginning with the lowest, and being made to pronounce 'one,' 'two,' and 'three,' in connexion [sic] with the names of the objects. So far he may be supposed to know the names of the different collections in the same way he knows the meaning of the word 'table'.[31]

De Morgan extended this process to objects out of sight for questions such as, 'If you have three apples and take away one, how many are left?' Finally, he removed the objects themselves from the reasoning and thus the child was left with a conception of an abstract number.

De Morgan advocated this same focus on aligning words with their mental conceptions in geometrical instruction. 'The first thing to be done is, not to *give* the notions attached to the words point, line, straight line, surface, and plane surface, for they exist already, but to take care that the ideas are attached to the right words'.[32] With proper instruction, either through teaching or careful texts, the student ought to gain knowledge of the proper first principles. Once there, it was only a matter of learning enough rules and logic to follow these initial principles to their logical and necessary conclusions.

5. Utilizing the impossible: arithmetic and the triumph of utility over clarity

Despite De Morgan's desire for his texts to move from clear premises to logical conclusions, many mathematical objects and concepts lack clear premises or easily defined foundations. Deceptively difficult issues, such as negatives and complex numbers, expose De Morgan's full faith in the power of reasoning, because De Morgan found a way to mathematically understand them even though they do have easily identifiable correlations in the material world. These are the subjects traditional texts

[30] De Morgan (1833a), p. 2.
[31] Ibid., p. 3.
[32] De Morgan (1833b), p. 37.

such as Bonnycastle's avoided and they reveal a great deal about De Morgan's pedagogical approach because they challenge the notion of mathematics as purely rational.

Rice has asserted that a potential response to these challenges is adjusting standards to expediently educate, even if it required a 'temporary sacrifice of rigour'.[33] A closer look, however, reveals no sacrifice of rigor, but rather that for clarity, De Morgan switched the focus from the erroneous mathematical conclusion to the logic of the assumptions and mechanisms of the problem. When a negative number or $\sqrt{-1}$ appeared as a solution in practice, the student should not accept this as accurate, but re-examine the problem itself. Despite their questionable foundation, De Morgan understood negatives and imaginary numbers through careful attention to the process used to derive them; that is, by careful examination of the initial postulates of and logic used in the problem.

The young De Morgan had a curious aversion to negative numbers, often attributed to his relationship with William Frend. These two met shortly after De Morgan came to London in the 1820s, and the elder Frend's opposition to negative numbers could have been a decisive influence on De Morgan.[34] De Morgan's mathematical treatises for the SDUK also dealt with these concerns, and show that in this particular context, he was able to assert a more nuanced approach to such foundational questions.

In his article for the *Penny Cyclopedia* entitled 'Negative and impossible quantities', De Morgan admits that others have found a foundation for negatives: Newton and Euler acknowledged the existence of quantities less than nothing. Often, these authors would use debits and credits to give this conception a physical equivalent.[35] Nonetheless, De Morgan's fundamental problem arose in his initial attempts to define number:

> The notion of number is suggested by repetition or succession; and it is customary to call the actual things repeated, considered as a collection, a concrete number; while the notion formed from comparing the collection with one of the things collected is called an *abstract* number. This abstract number arises from repetition of objects, in which the attention is directed to the repetitions as repetitions, and not to the objects as distinguished from other objects. It is therefore a number of times, not a number of things.[36]

[33] Rice (1997), p. 118.

[34] Pycior (1983), p. 214.

[35] De Morgan (1840a), pp. 130–137.

[36] De Morgan (1840b), pp. 363–364. De Morgan then utilizes the Greek definition that '*number* must *be more than one*', which seems to create some problems, especially considering the positive fractions. Later, he explains fractions as compositions of numbers even though they themselves are also numbers. De Morgan is, however, unable to justify negatives using his definition of number.

This definition of number (and even more so, of abstract number) by repetition prohibited the existence of negative numbers—there is no such thing as negative repetition, or even reverse repetition. Frend's objection stemmed more from his attempt to remove all mysteries from mathematics, a response to the mathematicians who used the lack of explanation for negatives and imaginaries as proof of a divine mystery. Frend completely rejected this Anglican mysticism in favor of the Unitarian focus on reason and evidence, just as he rejected the 'mystery' of the Trinity.[37] By De Morgan's and Frend's standard of clear definitions which were logically followed to conclusions, the negatives would be rejected, yet De Morgan had to explain their continued existence and seeming usefulness.

De Morgan saw a negative solution as posing a challenge to the terms of the problem itself. For instance, if 3–8 were the answer to a problem, then 'it would denote either that there was some absurdity inherent in the problem itself, or in the manner of putting it into an equation'.[38] Regardless, it is possible to incorporate 3–8 into the problem itself, because we can consider it as -5 connected with a larger number, which gives a positive result after subtraction. That is, you can perform operations with meaningless symbols because 'our present object' is to 'watch the results'.[39] This justification, taken from *On the study and difficulties of mathematics*, is part of an extensive section where De Morgan demonstrates that negatives simply indicate the postulates of the problem itself are incorrect, or that the interpretation of the solution is mistaken. If one were to get a negative answer when one were looking for velocity, it could mean that the positive direction of travel is incorrect, for instance. 'A negative solution', for De Morgan, 'indicates that the very nature of the answer is the very reverse of that which it was supposed to be in the solution'—a problem, but addressable if a student examines the premises of the question.[40]

What was more problematic for De Morgan were the fundamental properties of the negatives. For instance, is 0 really larger than -2? Or is $6 - 6$ greater than $6 - 8$? He says these questions are:

> Absurd... if we continue to mean by the terms greater and less, nothing more than is usually meant by them in arithmetic; but in extending the meaning of one term, we must extend the meaning of all that are connected with it, and we are obliged to apply the terms greater and less in the following way. Of two algebraical quantities with the same or different signs, the one is the greater which, when both are connected with a number numerically greater than either of them, gives the greater result.[41]

[37] De Morgan himself argued this point in an obituary of Frend: De Morgan (1842), p. 146.
[38] De Morgan (1830), p. 34.
[39] Ibid., p. 34.
[40] Ibid., p. 40.
[41] Ibid., p. 49.

Thus, one only needs to consider $20 - 6$ and $20 - 8$ to determine that $-6 > -8$. This nicely circumvents the problem of actually determining 'how negative' by substituting 'what effect on positive numbers does it have?' That is, a process is substituted for a property. Once again, the focus for De Morgan returns to the process, not to the result. This is the same underlying technique that allows -5 to be used for $3 - 8$ in a problem, as both expressions require the subtraction of five. De Morgan explains the conceptual contradiction with negative numbers by interpreting them as representative of problems in method or initial postulates.

De Morgan utilized the rigor of geometry to justify and clarify his understanding of negative and complex quantities. Extending the meaning of the signs to spatial quantities (length, direction, etc.) determined whether any result was 'real'.[42] In his article on 'Negative and impossible quantities', he describes a system that would provide meaning for these numbers by assigning negativity to be the opposite direction, such that $-a$ and $+a$ would have the same magnitude in opposite directions. Similarly, $\sqrt{-1}$ is a line equal in length to the unit line but inclined at a right angle. Thus, using geometry, one can put a rational interpretation on what appear to be difficult concepts. To conclude, he counters any potential objectors:

> If anyone should object that it is founded on geometry, we answer that it is not so much founded on geometry as applied to it. The symbolical algebra, which we draw in the first instance from arithmetical suggestions, and afterwards cut loose, so to speak, from that science, founding it upon purely symbolic definitions, is applied to geometry, because in the latter science, and in the latter only, do we find notions of magnitude, the different affectations of which are sufficient to supply rational meaning to all its symbols.[43]

Thus, he relates the troublesome quantities to what he thought of as the most fundamentally secure field of mathematics. The relationships in geometry were more 'transparent' and thus could serve as a model.

The parallels to geometry underscore the fact that De Morgan's primary objective was to ensure clear initial steps and logical development. By focusing on this aspect, De Morgan questioned many of the traditional and prevalent pedagogical techniques. When clear principles were difficult to find, as in negatives and imaginaries, he turned to the process to find errors in initial postulates and reasoning; clarity was still the focus, and a lack of clarity in reasoning or assumption caused the confusion. Thus, he could use 'impossible' quantities once they were carefully explained and logically handled. This focus on clarity and logic is exactly the kind of pedagogy that Brougham wished to inculcate generally across the nation. In his publications for Brougham and the SDUK, De Morgan's treatment of mathematics was quite differ-

[42] De Morgan (1840a), p. 135.

[43] Ibid., p. 136. Although not referenced, this exposition is very similar to the treatment given by Warren (1828). Furthermore, this was by no means a new idea; d'Alembert had used the analogy with geometry as early as the eighteenth century. For a summary of this French tradition, see Daston (1986), pp. 269–295.

ent from the prevailing methodologies of his day. Now we turn to how De Morgan's focus on reason played a role in mathematics more broadly, and finally we investigate how De Morgan's approach fitted in with the Society's moral mission.

6. De Morgan and the place of reason within mathematics

We have seen that the texts for the SDUK repeatedly emphasized the importance of a logical and reasonable foundation for mathematics. The most self-conscious author in this regard was certainly De Morgan, but all the authors were caught up in a characteristically British desire to ground mathematics in 'reality' and logic. Mathematics occupied a special role in British society, especially distinct from its role in France. In the latter country, the tradition of Gaspard Monge and descriptive geometry was giving way to Augustin Cauchy and a new focus on the precision of definition and symbol. For Cauchy, symbols lost external meaning and became tools for rigid mathematical manipulation, devoid of easy interpretation.[44]

The English, however, maintained the idea that the study of mathematics in a liberal education was a way to train gentlemen. This meant rejecting the conception of the subject as one whose direct applications were of paramount importance. William Whewell, in his *Thoughts on the study of mathematics as part of a liberal education*, explained this focus on mathematics as a way to help 'students become more fully formed human beings'.[45] Mathematics essentially trained the young minds to be rational, fulfilling the university's obligation to form the character of youth.[46]

According to this logic, the applications and the mathematical details were not crucial. Although conjoining the Cambridge-based Whewell and the London-based SDUK seems artificial from a historical perspective, as a Cambridge tutor and influential instructor of De Morgan and others, Whewell encouraged the British role of mathematics to extend beyond practical application. De Morgan himself, although he disagreed with some points of Whewell's conception of rigorous mathematics, supported this evaluation:

> Those to whom the mathematical sciences are taught as aids to the power of distinguishing truth from falsehood, logic from fallacy, the exact consequences from all incorrect inference, very many times exceed in number those who only wish for an instrument to be used in the study of physics and the arts of life.[47]

[44] Richards and others have published accounts of the rise of Cauchy and its implications, most notably in the movement away from the ideals of the enlightenment. See Richards (1991), p. 313. Most general texts on the history of mathematics will support her analysis; see especially Rouse Ball (1888), Grattan-Guinness (1980), Grattan-Guinness (1990). For the relationship of British science to Scottish philosophy, see Olson (1975).

[45] Richards (1991), p. 308.

[46] Whewell (1836), p. 39.

[47] De Morgan (1841), p. 53.

De Morgan's view was that mathematics is crucial as a skill, not as a body of knowledge to be learned. Matthew Jones has written about Descartes's emphasis on the practice of geometry as a means to spiritual understanding, and De Morgan seemed to believe a similar advantage could be gained from mathematics generally.[48] Reasoning skills for De Morgan were not innate, but needed to be learned: 'It is, therefore, necessary to *learn to reason* before we can expect to be able to reason, as it is to learn to swim or fence, in order to attain either of those arts'.[49] His treatises aimed to teach students to reason, a skill required for becoming an educated English gentleman.

De Morgan's educational focus took precedence in his publications and mathematical expositions. Both the French and English mathematicians were influenced by their dual role as teachers, but men such as Cauchy were famous for their incomprehensibility while De Morgan was praised as a superb educator. The overarching concern for De Morgan was the clarity of the initial postulates and the logic of the development, not the precision of the mathematical terms themselves. De Morgan and other Britons were certainly attempting to develop mathematics as a field, but they propagated mathematical knowledge with the recognition that most students did not study it for that purpose alone.

De Morgan's choice to provide a rational foundation for mathematics was fundamentally related to his own educational background. By the 1820s, the Cambridge University curriculum was firmly rooted in mathematics, with collegiate honors determined by the annual Mathematical Tripos exam. William Whewell supported the Tripos on the basis that the best mathematicians would be the best lawyers and doctors, even if they never used the mathematics after university. The skills mathematics promoted—logical reasoning, sound argumentation, and careful thought—made it the optimal subject for a liberal education. Fundamentally, Whewell and De Morgan believed in the power of mathematics, if studied well, to shape boys into good men.

If the SDUK authors were so concerned about reason, how did such a concern manifest itself in their treatises? This was a bi-directional process. On one hand, De Morgan and the SDUK general committee propagated mathematics in such a way as to make the mathematics itself rational. When faced with controversial or little-understood topics, they attempted to rationalize them; clarity for students was crucial and it was repeatedly emphasized that only by studying mathematics as an exercise in pure reason would one gain the desired benefit. In this process, the mathematics was shaped by the author to fit his own conceptions of rigor and validity.

On the other hand, conceptions of God and morality were accessed and understood through a mathematical framework. Most fundamentally, De Morgan believed that God had bestowed the power of reason to distinguish men from beasts. Mathematical study thus promoted development of this particular conception of the role of humans in the world. Mathematics was projected onto and also

[48] Jones (2001), pp. 40–71.
[49] De Morgan (1830), p. 3.

shaped the conceptions of morality and religion that flowed out of the SDUK. The interweaving of morality and mathematics is the subject of the final section; the sense in which mathematics is shaped as a moral activity is evident through a close examination of the transformations in De Morgan's algebra.

7. Algebra from base numbers to pure reason

De Morgan's understanding of the meaning (and foundations) of algebra shifted throughout his life and his algebraic publications for the SDUK are indicative of the way he framed his mathematics based upon a conception of the subject as fundamentally rational. Initially, he saw algebra as a system substituting general symbols for arithmetic numbers and processes, but he gradually came to remove all attachment of numbers from algebra. While early in De Morgan's career, algebra was grounded by the properties of numbers, by the end, multiple algebraic systems might legitimately exist, each with its own meanings and interpretations. I will demonstrate how he actually shaped his algebraic beliefs for readers in the context of the SDUK.[50] A critical moment in the algebraic output of De Morgan occurred in 1835: in that year he published a review supporting George Peacock's vision of algebra removed from numerical analysis and grounded in reason, a vision very different from De Morgan's previous writings.

This transformation hinged on the evaluation of the equation $(a^2 - b^2) = (a + b)(a - b)$. If $a > b$, then this statement is true in both algebra and arithmetic, but if $b > a$, the situation grows more complex. De Morgan explicitly rejected the value $(a - b)$ when $b > a$, saying it has 'no imaginable existence'.[51] Yet Peacock, in his *Treatise on algebra*, developed the concept of 'the principle of equivalent forms'. This explained the relationship between arithmetic and algebra in a way that allowed maximum flexibility to the mathematician. A concise version of it reads: 'Whatever equivalent form is discoverable in arithmetical algebra considered as the science of suggestion, when the symbols are general in their form, though specific in their value, will continue to be an equivalent form when the symbols are general in their nature as well as their form'.[52] Under this system, one could take an equation, say, $(5 - 3)(5 - 3) = 5^2 - 2(5)(3) + 3^2$, change it to $(a - b)(a - b) = (a^2 - 2ab + b^2)$, and grant the validity of the expression for any a and b. The symbols lose what Ernest Nagel has called '*extrasystemic* reference, concerning which questions of truth and falsity are significant' in favor of a 'system of marks with no such *explicit*

[50] The general topic of De Morgan's work within algebra has been extensively covered by both Helena Pycior and Joan Richards, and I will focus solely on how this transformation was brought about by the SDUK's focus on reason and logic as the ultimate foundation for mathematics. See Pycior (1982), Pycior (1983), and Richards (1987).

[51] De Morgan (1840a), p. 131. That is, if $b = 5$ and $a = 3$, $(9 - 25)$ and $(8)(-2)$ cannot be equal because neither exists. But if $a = 5$ and $b = 3$, then $16 = 16$, and the equation is valid.

[52] Peacock's 'Report on the recent progress and present state of certain branches of analysis', quoted in Richards (1987), p. 14.

extrasystemic interpretation', where only questions of validity, but not of truth, have meaning.[53] When algebra is in its most general form and symbols can have meaning and tests of validity unrelated to arithmetic, logic and reason form the only basis of the system.

De Morgan's review of Peacock's work in 1835 marked a shift toward algebra grounded in reason, based on his work for the SDUK. In the midst of writing for them, De Morgan's decision to adopt a new meaning for algebra demonstrates the degree to which he shaped his mathematics to the ends of the Society. By 1837, De Morgan expressed the ways in which algebraic reason can triumph over numerical calculations: 'An expression may preserve an algebraical identity with its development in cases where arithmetical identity is entirely lost... a development may present an intelligent result, in cases where the original expression altogether loses meaning'.[54] He continues with an example of the infinite sum $(1 + 2 + 4 + 8 + \cdots)$ to illustrate how algebra and arithmetic can provide two contradictory interpretations of one situation. De Morgan asserted that this seemingly infinite sum is really equal to -1 because we can consider the algebraic sum $P = (1 + x + x^2 + \cdots)$. Using simple symbolic manipulation, he found that if $P = (1 + x + x^2 + \cdots)$, then $xP = x(1) + x(x) + x(x^2) + \cdots$ Next, De Morgan asserted that $P = 1 + xP$ and solved for P to find $P = \frac{1}{1-x}$. De Morgan then transformed P into an arithmetic expression by setting $x = 2$, suggesting the sum $(1 + 2 + 4 + 8 + \cdots) = P$ and $(2 + 4 + 8 + 16 + \cdots) = 2P$. The addition of one to the latter series is equivalent to P, so $P = 1 + 2P$, as we would expect since $P = 1 + xP$ and $x = 2$. Combining $x = 2$ with the algebraic result that $P = \frac{1}{1-x}$, De Morgan concluded that $P = -1$. A comparison of the algebra and arithmetic suggested $P = -1$ and $P = (1 + 2 + 4 + 8 + \cdots)$. This leads to the paradox of an infinite series of positive numbers adding up to a negative number: $(1 + 2 + 4 + 8 + \cdots) = -1$.

De Morgan vigorously defended the purely algebraic result that $1 + 2 + 4 + \cdots$ sums to -1 as late as 1844.[55] In 1839, De Morgan was already asserting that 'algebra was sometimes called universal arithmetic, but the phrase never became general, owing to its being obvious to those who studied algebra, that arithmetic, however, general its symbols may be, is only a part of algebra'.[56] The distinction between simply substituting variables for numbers and working with the variables themselves is, according to De Morgan, the realm of 'interpretation'. Algebra was differentiated from arithmetic because in the former one can abstract a numerical fact and then manipulate and interpret it without regard for numerical values or properties.

De Morgan's acceptance of an abstract algebraic system with no extrasystemic reference introduced only questions of validity and logic, not those of truth. This new conception of algebra is wholly consistent with his general attempt to make

[53] Nagel (1935), p. 451; emphasis in original.

[54] De Morgan (1837), p. 445.

[55] For an analysis of how this problem relates to De Morgan's place within English mathematics, see Richards (1987), pp. 26–27.

[56] De Morgan (1839), p. 12.

his mathematical treatises for the SDUK fundamentally logical. Sir William Rowan Hamilton noticed this when he remarked that Peacock (and later, De Morgan) 'designed to reduce algebra to a mere system of symbols and *nothing more*; an affair of pothooks and hangers, of black strokes upon white paper, to be made according to a fixed but arbitrary set of rules: and I refused, in my mind, to give the high name of *Science* to the results of such a system'.[57] De Morgan was more flexible in his criteria for useful sciences than Hamilton: algebra was a useful skill for students and thus could be studied and mastered as a way to improve the intellect, even if there were not a necessary connection to arithmetic. De Morgan used the same image of hooks, when he explained that, 'On the absolute substantive *reality* of all the primary truths of mathematics I have never had any doubt: but I have an idea that different people hold them by different hooks'.[58] De Morgan claimed that there is substantive truth to mathematics, beyond manipulation of symbols, but people can reach it from different applications of reason.

In this same review of 1835, De Morgan also comments on John Warren's algebraic work, which proposed that geometry, not arithmetic, could serve as a way to connect algebra to concrete examples. Warren's *Treatise on the geometrical representation of the square roots of negative quantities* might have shown the possibility of developing multiple models of symbolical algebra to De Morgan.[59] On this possibility, De Morgan mused, 'Whether it might not be possible so to vary the meaning of the signs, as to make an entirely different algebra, which should nevertheless present exactly the same theorems in form as the old one, the forms having different meanings'.[60] In his article on 'Negative and impossible quantities', De Morgan explicitly supported this idea of multiplicity:

> It is not true that there is only one set of meanings under which the fundamental relations of algebra are truths, for three sets have been already alluded to in this article, namely, the common and limited arithmetical meanings, the extensions under which the difficulties of the negative sign disappear, and the geometrical meanings last described.[61]

This was the algebra alluded to earlier, removed from numerical meaning and shaped to exist as a well defined logical system; there was no particular virtue within the meaning of symbols, but the true emphasis was on the logical and reasoning skills required to understand the system.[62]

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[57] Quoted in Bloor (1991), p. 218; emphasis in original.

[58] Letter from De Morgan to William Whewell, 4/30/1844, Whewell Archive, WL.

[59] Pycior (1983), p. 220; see also Warren (1828).

[60] De Morgan (1835), p. 108, quoted in Pycior (1983), p. 220. Peacock himself was not willing to extend algebra to such an extent, saying that 'it would be a serious mistake, therefore, to suppose that such incidental properties of quantities affected by such signs constituted their real essence'. For him, it was simply an extension of geometry, not an explanation of algebra. Quoted in Bloor (1991), p. 221.

[61] De Morgan (1840a), p. 133.

[62] De Morgan (1835), p. 103.

8. Mathematics as the means to religion and morality

This emphasis on logic and reason was not only consistent with the moral mission of the SDUK but also crucial for its success. At first glance, the Society maintained a paradoxical position: they would affect morality while producing no major writings on personal moral development or revealed religion. This is not to say religion was not important—in early nineteenth-century England, religion still structured society to a large degree. All of the SDUK founders were religious and most were educated at Anglican Oxbridge, but many also supported secular education. Even if they had wished to include religious principles, the diverse group of Anglicans, dissenters, and Jews probably could not have agreed on a set of particular truths. To see how the Society aimed to improve the moral and religious character of the nation we must broaden our scope beyond De Morgan and locate his mathematical treatises within the concurrent discussions concerning the relationship between religion and the sciences.

The mathematical texts of the Society were fundamentally the most useful for the improved progress of the laboring classes because the skills they promoted advanced moral reasoning ability, even if no principles of religion were in the treatises. There were three different methods suggested by Brougham and prevalent at the time for the moral improvement of the population through the study of mathematics, all based upon a particular conception of God as infallible, whose works on earth reflect that perfection. The first was that the study of mathematics was thought to reveal to humans the perfect, and thus the divine. This occurred both by the development of morality based on utilitarian principles, that is, upon a calculus of right and wrong, and by the perfect nature of the mathematics itself. Second, in the pursuit of happiness, we act according to what gives pleasure, and having logical reasoning skills will not only assist in the normative evaluation of different actions but also give pleasure directly to the exerciser. Finally, William Whewell and others argued that God reveals his glory through inductive reasoning, and thus the study of mathematics (properly presented) and the refining of logical modes of inquiry will allow us to appreciate and understand God's great creations and draw us nearer to the Creator himself. Examining these interwoven threads provides a backdrop necessary for understanding how De Morgan's mathematical treatises functioned in the larger debate over what constituted useful knowledge.

8.1. Mathematics, utilitarian choice, and the pursuit of perfection

Brougham's preliminary treatise for the SDUK, the *Discourse of the objects, advantages and pleasures of science*, underscored the importance science and mathematics would play in the Society's objectives while also illuminating the benefits of such studies. The bulk of his argument was not based upon the direct application of science, although he did acknowledge such benefits. Instead, his primary argument was that we gain pleasure through the study of the natural sciences, especially mathematics. Every man, regardless of class, has the natural capacity to be pleased by mere knowledge of mathematics.

> Hence to follow a demonstration of a grand mathematical truth—to perceive how clearly and how inevitably one step succeeds another, and how the whole steps lead to the conclusions—to observe how certainly and unerringly the reasoning goes on from things perfectly self-evident, and by the smallest addition at each step, every one being as easily taken after the one before, as the first step of all was, and yet the result being something not only far from self-evident, but so general and strange, that you can hardly believe it to be true, and are only convinced of it by going over the whole reasoning—this operation of the understanding, to those who so exercise themselves, always affords the highest delight.[63]

Exercising mathematics skills gives pleasure related implicitly to a conception of virtue and the good. The late eighteenth and early nineteenth centuries were the high water mark of rational systems of morality, perhaps most successfully embodied by the utilitarians. The men often cited as landmark authors of utilitarianism are Jeremy Bentham and John Stuart Mill, both of whom had associations with the SDUK. The former, one of the founders of the University of London, was in regular contact with Brougham, while the latter's father was one of the original founders of the SDUK. Both Bentham and Mill believed education was necessary for enlightenment, for only through the propagation of a rational scientific education would the human condition be improved.[64] Further, their shared belief that every decision embodied an analysis of pain and pleasure required general acceptance of the power and utility of mathematical and logical reasoning.

In Brougham's mind, mathematics served a dual purpose: it was both a crucial mechanism behind utilitarianism and a pursuit that would itself be valued because of the inherent pleasure. In his conception, morality arises from the ability to rationally discern the pleasure derived from an act, and Brougham saw no activity that encouraged rationality and pleasure together more than the study of mathematics. Contextually, this argument might seem to ring hollow; if the intended readers really were the working classes of the country, it is doubtful that they would be able to derive the same kind of pleasure from mathematics as Lord Brougham. Some might even see this as evidence of the great divide between the reformers and those to be reformed, as something that perhaps escalated working class disenchantment that would take form in the Chartist movement.[65]

What is most crucial for us, though, was the way this moral conception supported Brougham's focus on mathematics. For him, the 'pleasures of Science go hand in hand with the solid benefits derived from it; that they tend, unlike other gratifications, not only to make our lives more agreeable, but better; and that a rational being is bound by every motive of interest and of duty, to direct his mind towards pursuits

[63] Brougham (1827), p. 175.
[64] Simon (1960), p. 144.
[65] For more information on Chartism, see Jones (1983).

which are found to be the sure path of Virtue as well as Happiness'.[66] De Morgan explicitly allied himself with Brougham when he justified mathematical study to his students at the University of London by asserting that the powers that 'will be required to distinguish between right and wrong, are not adequate to this purpose, unless a habit of using them rightly have been previously formed'.[67] Constant mathematical study, of course, was his solution. De Morgan said these benefits had 'indirect utility', and that it was 'generally admitted' that whenever 'the strengths of sound reasoning [are] clearly appreciated... the preparation of mathematical studies is useful in the highest degree'.[68] Without explicitly linking their positions to any one moral system, Brougham and De Morgan connected moral structures with the pleasures of science, and the study of mathematics with the improvement of humanity.

Brougham also justified the study of mathematics by demonstrating that the results of mathematical investigation are perfect and without challenge. For him, all perfect things must be works of God, because God is the most perfect being; thus, the study of mathematics brings one closer to understanding the glory and wisdom of God. Mathematical truths for Brougham, and for many academicians in the early nineteenth century, 'are *necessarily* such,—they are truths of themselves, and wholly independent of facts and experiments,—they depend only upon reasoning; and it is utterly impossible they should be otherwise than true'.[69] Mathematics builds on material foundations to provide eternal truths, according to Brougham, and to offer the reassurance that God bestowed in us the power to appreciate such truths. 'No man, until he has studied philosophy, can have a just idea of the great things for which Providence has fitted his understanding... there soon arises a sense of gratification and of new wonder at perceiving how so insignificant a creature has been able to reach such a knowledge of the unbounded system of the universe'.[70] That is, mathematics bridges the gap between our partial understanding and the full knowledge of the divine. Developing mathematics and reason allows us to see, to follow, and to appreciate the works of God.

8.2. Inductive logic and the progress to the eternal

The final justification given for the propagation of mathematics in place of revealed religion was the subject's emphasis on reasoning, which was crucial both to understanding the workings of the universe and to appreciating and adoring the handiwork of its Architect. Henry Brougham elucidated this justification most clearly in his *Discourse of the objects*, *advantages*, *and pleasures of science*:

[66] Brougham (1827), pp. 182–183.
[67] De Morgan (1828), p. 9.
[68] Ibid., pp. 43–44.
[69] Brougham (1827), p. 36; emphasis in original.
[70] Ibid., p. 179.

> It is through this kind of reasoning, step by step, from the most plain and evident things, that we arrive at the knowledge of other things which seem at first not true, or at least not generally true; but when we do arrive at them, we perceive that they are just as true, and for the same reasons, as the first and most obvious matters.[71]

Mathematics transforms agreed-upon axioms to reveal the more sublime reality underlying our own understanding. William Whewell differentiated his beliefs slightly from those of Brougham by asserting that mathematics should be taught alongside religion; education needed a strong moral and religious element. Brougham, along with William Hopkins, saw the study of advanced mathematics as a 'virtuous activity in its own right' because it revealed divine perfection and provided useful applications.[72] Furthermore, Brougham and De Morgan saw elementary mathematics as useful because of its focus on inductive logic.

The emphasis on reason is apparent in the transformation of arithmetic to algebra in De Morgan's works. Algebra was initially portrayed as universal arithmetic abstracted until a state existed where the forms are 'perfect', but have truth independent of numbers. Similarly, we aim to use earthly facts to abstract the meaning of the divine. Once abstracted, human concerns of truth and falsehood have no meaning based on earthly standards. Whewell described this transformation by proclaiming, 'We stand upon the visible things of this world, and lift our eyes to the invisible things of his eternal power and godhead'.[73] Whewell's views, largely similar to Brougham's, are worth investigating because of his continued support of mathematics within the curriculum as well as his influential role promoting mathematics as useful knowledge.

For Whewell, studying the sciences was more than simply a means of learning a particular structure of reasoning; inductive reasoning was one of the primary ways for us to discover divine truths. The reasoning itself was divine, not just the abstracted product. When joined by study of scripture, it fostered the true knowledge of God. Significantly, he elucidated his philosophy in church sermons, a very different context than the ones in which Brougham and De Morgan worked. In a religious age, Whewell's support of mathematical education took on a particularly significant meaning in this localized context. In his sermons, he berated those who have lost sight of '*His* hand and *His* mind' in the pursuit of 'novelties', an error made far more grievous when one considers that Whewell believed the position of scientists brought them 'closer to the Deity than others are', even if they were not wearing the 'splendid garb' of priests.[74] Mathematicians had a particular obligation to reveal the structure of the universe, and the primary means of accomplishing this in Whewell's view was through inductive logic, lifting our eyes from the visible to the invisible. This process is hardly dry or devoid of emotion—'what we should strive to arrive at is not an

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71 Ibid., pp. 37–38.
72 Warwick (2003), p. 181.
73 Whewell, Sermon, 11 February 1827, Whewell Papers, WL.
74 Whewell, Sermon, 4 February 1827, Whewell Papers, WL.

interference of the reason alone but of the heart; our end must be not to convince only but to persuade; to lay a foundation—not for momentary assent, but for a settled habit of mind'.[75] Based on Whewell's imperative, De Morgan had moral justification to continually emphasize the importance of rationality.

In the notes for his sermons, Whewell repeatedly emphasizes the importance not of discovering useless nuggets of information, but of progress toward meaningful—that is, divine—discoveries. More broadly, his primary philosophical contribution was the recognition that properly logical scientific structures gradually uncover and order conceptual reality.[76] Making proper scientific study a habit will create religious and knowledgeable men, and thus the propagation of mathematics will lead to a more devoted population as well as one better versed in the divinity.

> As we mount up to truths more and more general, we approach towards those acts of his volition which bind, as it were, all things in an indissoluble chain. The laws, then, which are the subject of our search, acquire as it were a majesty not only from their sway and universality, but from their origin and author. They govern all things because they connect all things with his will.[77]

Whewell's God is rational, and induction is the way to understand His creation. Brougham and Whewell both believed that the devoted study of mathematics would bring pleasure and morality to the participant, and that the rationality of the results of such study would connect us with the will of God.

This was hardly the only view of the proper ways to encourage religious development. The sermons by Whewell were in response to one by Hugh James Rose rejecting mathematics as the basis for a liberal education. For another critic of the Whewellian system, it was not the substance of mathematics which bothered him, but the separation of religious knowledge from 'useful' knowledge that rendered the SDUK's efforts impotent. In a sermon delivered in 1841 on the 'Worth of knowledge', Julius Charles Hare repeatedly calls for education based on the 'organic system of truth',[78] not the propagation of scattered nuggets of knowledge:

> Therefore, we are not to let our children be trained in secular knowledge, as a preparation for religious knowledge, being certified by the history of Solomon, and by ever-renewed experience in all classes of life, as well as by our Lord's words, that such a preparation may often prove a grievous hindrance. Nor are we to recognize a separation and division between secular knowledge, as it is called, and religious knowledge, and to halve the souls of our children between the two.[79]

[75] Whewell, Sermon, 11 February 1827.
[76] Richards (1987), p. 20.
[77] Whewell, Sermon, 11 February 1827.
[78] Hare (1858), p. 232.
[79] Ibid., p. 231.

From his point of view, exclusion of religious knowledge from the SDUK's publications rendered them useless, despite the rhetoric that inductive logic could bring better understanding and appreciation of the great Architect of the universe. He concluded that science must always be yoked to religious and scriptural knowledge.

In a society where most people spent hours every Sunday listening to sermons, these texts provided epistemological contexts for the work of the SDUK. Furthermore, the prominent individuals giving sermons at Cambridge, working within the SDUK, and employed in London scientific and political circles were in constant dialogue on these issues. The SDUK's treatises functioned to produce useful knowledge in the context of secular education, modeled after the writings of Brougham and Whewell. Inevitably, critics such as Rose and Hare challenged such a system, and their opinions help frame the work of the SDUK within the larger epistemological context of nineteenth-century England.

9. Conclusion

In the last section we saw how De Morgan's treatises, when considered alongside the arguments of men such as Brougham and Whewell, were central to a particular conception of moral development. His mathematical work for the SDUK thus had a dual role: it shaped mathematics as a rational and logical activity and it functioned as a moral instrument in the development of reason, intending to inculcate useful skills to the English populace. In these treatises, De Morgan developed a new pedagogical treatment of mathematics, one with the student as its focus, which countered the prevailing trends in nineteenth-century mathematical education.

Significantly, this new treatment actually influenced the way he taught mathematics, especially in his analysis of negative numbers and his approach to algebra. In both areas, his shift to a system based on logic and clarity underscored the degree to which his pedagogical beliefs influenced his intellectual writings. In turn, his treatises served the moral purpose of the SDUK though the focus on reason and induction. One could argue, a bit more speculatively, that the ever private De Morgan was expressing a Unitarian belief in reason through his mathematical writings; for him, mathematics and religion were intricately linked even if his texts lacked the clear references to religion present in those of Brougham and Whewell.

The SDUK endeavored to improve morality through non-religious texts; they believed secular education, rather than treatises on revealed religion, would help the religious state prosper. Because of De Morgan's new pedagogical approach to mathematics, his treatises for the Society functioned as the link between the religious and secular worlds. It was in this context of moral improvement that De Morgan attempted to radically change the course of English mathematical education in the first half of the nineteenth century.

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