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# Knowing By Number: Learning Math for Thinking Well

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If late modernity's central measure of cultural import is the viral YouTube video, then the Common Core mathematics standards have proved infamous indeed. Karen Lamoreaux's 2013 appearance before the Arkansas Board of Education has been viewed over three million times and is evocatively titled, "Arkansas Mother Obliterates Common Core in 4 Minutes!"[1] Lamoreaux's gripe was not with the standards per se, but rather with the method of problem solving the standards apparently promoted. Her fourth grader's class of eighteen students was presented with this conundrum: if they "counted around" by some particular number, and ended on ninety, with what number did they count around?[2] Lamoreaux points out that she would have solved it by asking herself what number times eighteen produced ninety, namely five. Two steps, and done. The students, apparently, were instead asked to draw eighteen circles, and then ninety hash marks, evenly distributed, to arrive at the answer of five per circle—a process that took a minimum of 108 steps. Surely this, Lamoreaux suggested, was decisive evidence of the curriculum's intellectual bankruptcy.

Nineteen seventy-two's version of the viral video was the "Letter to the Editor," and that autumn the Washington Post received an astoundingly high number of letters concerning an article that expressed a strikingly similar complaint. James Shackelford, PhD, a chemist with the Environmental Protection Agency, found that he did not understand the problems in his daughter's fourth grade math assignment, taken from one of the novel "new math" textbooks. Far worse, he found neither his daughter nor any of her friends knew the answer to his question, "What is eight times nine?" from memory. The Post featured a front-page story on his complaints, the upshot of which echoed Lamoreaux's: surely, Shackelford claimed, his experience was decisive evidence of the curriculum's failure.

Both episodes point to the seemingly inescapable conclusion that math class is about more than just reaching the "right" answer. You also have to get it in the "right" way. Many fields, after all, convey useful knowledge—astronomy, geography, home economics—but only a few are said to be "good for the mind," training students to face complicated problems generally. For millennia, mathematics has been thought of as the discipline that disciplines—putting the mind on the right track and training rational judgment. This assumption is still alive and well in the twenty-first century in the way most people talk about intelligence—a teenager who's a great mathematician is a genius, but a teenager who's a magnificent poet is, well, just a poet. As those applying to enter American graduate schools in English or social work quickly realize, the required Graduate Record Exam still demands mathematical competence, not knowledge of literature or psychology.

Mathematical practice is both a resource for and model of rigor, precision, proof, and certainty. In one of Plato's dialogues, Socrates shows Meno how even an uneducated slave "already knows" how to construct a specific sort of geometric figure. Socrates concludes that the slave had "true opinions on a subject without having knowledge." That is, mathematical knowledge was not from "teaching but from questioning"—geometry class does not convey knowledge but reveals true knowledge within all of us.[3] Christians, too, heralded mathematics' close relationship with reason—that God had written the "book of nature" in the language of mathematics, legible only though the application of uniquely human intelligence. The eighteenth-century mathematician Jean-Etienne Montucla declared that a well-done history of mathematics "could be looked upon as a history of the human mind, since it is in this science more than all others that man makes known the excellence of the gift of intelligence which God has given him to raise him above all other creatures."[4] And, in the nineteenth century, university positions in mathematics were doled out in part on the basis of what kind of mathematical instruction might be most helpful for young minds. When Sir William Hamilton wrote in support of Duncan Gregory's case for the University of Edinburgh Chair of Mathematics, he did so on the basis of Gregory's preference for geometry over algebra: "The mathematical process in the symbolical method [i.e., the algebraic] is like running a rail-road through a tunnelled mountain; that in the ostensive [i.e., the geometrical] like crossing the

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[1] "Arkansas Mother Obliterates Common Core in 4 Minutes!," *YouTube*, accessed April 25, 2016, https://www.youtube.com/watch?v=wZEGijN_8R0.

[2] "Counting around" means counting in multiples; counting around by seven would mean counting {7, 14, 21, . . .}.

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[3] Plato, "Meno," in *The Collected Dialogues of Plato Including the Letters*, ed. Edith Hamilton and H. Cairns, trans. W. K. C. Guthrie (Princeton, NJ: Princeton University Press, 1961): 353–84, on 370.

[4] Galileo's Assayer contains the best-known pronouncement that the "book of nature" was written in the language of mathematics: Stillman Drake, ed. and trans., *Discoveries and Opinions of Galileo* (New York, NY: Anchor Books, 1957), 238. Montucla quoted in Joan L. Richards, "Historical Mathematics in the French Eighteenth Century," *Isis* 97 (2006): 700–713, on 707. Similar assertions are made elsewhere, e.g., Lancelot Hogben, *Mathematics for the Million: A Popular Self Educator* (London: George Allen & Unwin, 1936), 34.

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mountain on foot. The former carries us, by a short and easy transit, to our destined point, but in miasma, darkness and torpidity, whereas the latter allows us to reach it only after time and trouble, but feasting us at each turn with glances of the earth and of the heavens, while we inhale health in the pleasant breeze, and gather new strength at every effort we put forth."[5] For Hamilton, as for many observers, the debate was not about which practice produced superior research, but which would promote the best form of reasoning.

Lamoreaux's complaint about the Common Core is just the latest iteration of a long history of worries that math class was failing to teach students to think in the right way. It was not simply that textbooks had factual errors or were pedagogically deficient. Rather, if mathematical truths are certain and reliable, then learning them entailed learning how to recognize certain and reliable knowledge more generally. As mathematics became a standard course of study over the last century, math educators have often encouraged this view that math education was general education. A 1923 Mathematical Association of America report on the proper aims and nature of mathematics instruction reasserted the centuries-old claim that "general mental discipline is a valid aim in education" and made the case that mathematics training might "transfer" between disciplines. At that time, the key was promoting a (non-technical) notion of mathematical functions to the expanding population of high school students, as the basis of "functional thinking" generally. In 1940, the Progressive Education Association emphasized a similarly transferable role for mathematics, no longer on the basis of functions but rather as a set of practical, specific skills for general education.[6]

Perhaps no historical example paralleled Lamoreaux's complaint so directly as did the new math reforms that prompted Shackelford's story in the Washington Post.[7] Though contemporaneous reforms also took place elsewhere at mid-century, the American reforms were in many cases the most explicitly political. The new math began as one of a number of National Science Foundation (NSF) projects to shape the nation's curriculum by engaging academic scientists to write model textbooks. Unlike the situation in many countries, American education was and is entirely decentralized, with states, local districts, and individual classroom teachers in charge of textbook selection and curricular decisions. With federal funding, the National Science Foundation provided time-and-a-half salary starting in 1958 for mathematicians to join with high school teachers and rewrite the nation's textbooks for grades nine through twelve. Eventually the School Mathematics Study Group, as the key NSF-funded initiative was called, expanded to all grades, as well as to "disadvantaged" and "slow" students. Though there were other reform efforts, only the School Mathematics Study Group had the imprimatur of official status, as it was supported by the American Mathematical Society, Mathematical Association of America, and the National Council of Teachers of Mathematics in addition to the NSF.

The NSF's curricular reform program had been initiated more than a year before Sputnik, but the launch of the Soviet satellite in the fall of 1957 meant that funding would be extraordinarily generous. In the two years after the launch, NSF spending on education projects increased over tenfold. The curricular program had widespread bipartisan support at first, with liberals pleased that money was being spent on education and conservatives happy that education funding was limited to mathematicians and scientists, rather than so-called progressive educators. Representatives from both sides of the aisle thought that model textbooks provided a mechanism for curricular change without impinging on states' and school districts' rights.

Moreover, the reforms were understood as addressing the problems of Cold War manpower shortages. For years, the NSF had published a Scientific Manpower Bulletin and fear mongers had consistently claimed (based on often spurious evidence) that the Soviet Union was producing far more scientists and engineers. As physicist Henry Smyth explained, scientists were "tools of war" to be "stockpiled" just "as we would any other essential resource."[8] Though the original mission of the Foundation was graduate research fellowships, then-director Alan Waterman soon realized that Congress would be far more willing to increase funding if pre-collegiate educational initiatives were included.

The politics of "scientific manpower" did not dictate what sort of mathematics to teach, only that it should be taught more. Many mathematicians involved with the School Mathematics Study Group, however, did have specific ideas about how math should be taught. They thought the discipline had for too long been portrayed as a "dead and completed subject that was embalmed between the covers of a textbook sometime after Sir Isaac Newton."[9] Moreover, teaching math as a "dead" subject also often meant teaching it as a set of facts to be memorized. This was not just stultifying, but also meant learning math in a way that could easily have been described as authoritarian. How might scientific education aid in the defeat of the Soviets if it were taught in such a way as to promote blind deference to authority? It was in this context that Howard Fehr, a School Mathematics Study Group supporter and one-time head of the National Council of Teachers of Mathematics, explained that "It is essential for the student of mathematics to have acquired the faculty of being able, by his own wit, to learn more mathematics, to solve new problems, to adapt his past knowledge to new knowledge and new points of view; and, above all, it is essential to him to have been liberated from the shackles of authority."[10] Likewise, a School Mathematics Study Group–allied reform effort

[5] George Davie, *The Democratic Intellect: Scotland and Her Universities in the Nineteenth Century*, 2nd ed. (Edinburgh: Edinburgh University Press, 1964), 127.

[6] The National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education* (Oberlin, OH: Mathematical Association of America, 1923), 8, 72, 90; Commission on the Secondary School Curriculum of the Progressive Education Association. *Mathematics in General Education: A Report of the Committee on the Function of Mathematics in General Education* (New York, NY: 1940), 11–12.

[7] The material of this section is discussed in much greater detail in Christopher J. Phillips, *The New Math: A Political History* (Chicago: University of Chicago Press, 2015).

[8] Henry D. Smyth, "The Stockpiling and Rationing of Scientific Manpower," *Bulletin of the Atomic Scientists* 7, no. 2 (1951): 38–42, 64. For the spurious claims of "shortages," see David Kaiser, "The Physics of Spin: Sputnik Politics and American Physicists in the 1950s," *Social Research* 73 (2006): 1225–52.

[9] School Mathematics Study Group, *Mathematics for Junior High School: Student's Text*, vol. 2 (New Haven, CT: Yale University Press, 1961), 545.

[10] Howard F. Fehr, "Modern Mathematics and Good Pedagogy," *Arithmetic Teacher* 10 (1963), 402–11, on 403–4.

described the goal of the new math curriculum in 1963 as, "whenever possible," giving students "some intrinsic criterion for deciding the correctness of answers, without requiring recourse to authority."[11] Mathematics for "thinking well," at the height of the Cold War in the late 1950s and early 1960s, meant mathematics for creatively, yet logically, getting to the right answer.

This, by and large, meant teaching mathematics as structure. That was ultimately the justification for the new math's emphasis on set notation, number lines, Venn Diagrams, and other notational trappings of logical—structural—reasoning. The point was not to promote internecine forms of mathematics, but rather to give children, starting in first grade, the ability to recognize mathematical correctness on their own. It was not the case that "anything went" mathematically—students still had to get mathematically correct answers. It was, however, important for the mathematicians and teachers involved with the School Mathematics Study Group to promote a form of mathematics that enabled the student to have confidence in the correctness of an answer by virtue of independent reasoning. Students "come to us," one teacher's commentary suggested, "with a miscellaneous hodgepodge of disjointed facts and pseudofacts" and it is the math teacher's job to "straighten out their ideas, to build a reasonable conceptual structure upon which they can hang new facts," enabling students "to distinguish between that which is significant and that which is not, and, perhaps most important of all, to understand how new knowledge is acquired."[12] Math class was a setting for epistemological training, not just for the transmission of useful skills.

The new math reforms had mathematical cover because the promotion of sets and logical reasoning fit well with the belief of many of the academic mathematicians involved that mathematics should be taught as pure reason, not necessarily connected to any particular applications. A number of the School Mathematics Study Group's writers were supporters of highly formal mathematical methods—a stance that certainly had detractors among their colleagues. The reforms had psychological cover in part through the support of Jerome Bruner's reading of Jean Piaget's stages of intelligence. Bruner claimed that modern—axiomatic and structural—topics in mathematics were most appropriate for teaching because "the sequence of psychological development follows more closely the axiomatic order of subject matter than it does the historical order of development of concepts within the field."[13] The new math had political cover because it was sold as "modern"—it was the sort of mathematics needed for a rapidly changing, increasingly technological world. The head of the School Mathematics Study Group had explained that specific skills were only of limited worth, for they might not be needed by the time students left school years later. The world was changing too rapidly. Teaching math for "intelligent citizenship"—as the reformers claimed to be doing—meant addressing the fact that no one would be able to predict the important and useful skills of the future.[14] The lament that a formerly static world was experiencing rapid change driven by mathematical and technological developments was an old trope, providing, for example, the basis for satire in Charles Dickens's 1854 novel Hard Times. However, when combined with the specific context of a democratic response to Cold War authoritarianism, it helps explain why the new math had such initial support.

Nevertheless, when the fall came for the reforms, it was steep and swift. Only a year or two after the School Mathematics Study Group's 1972 end, condemnations of the new math arose all over the country—extending far beyond Shackelford's complaint in the Washington Post. States across the country began pushing "traditional" or "back to basics" education in lieu of the new math reforms. A seemingly endless account of test-score declines provided a patina of credence to the idea that the reforms were hurting children's arithmetic ability, though the causal connection was dubious at best.

Critics of the new math in the 1970s were no less convinced than new math's proponents had been that learning math meant learning to reason. It was just that their view of the intellectual discipline that math class should promote had changed. In an age of the Vietnam War and the Watergate scandal, of increasing skepticism toward elite, top-down programs, and of a renewed emphasis on decentralized, local control, the new math reforms had the politics of knowledge all wrong. The strange notation and nomenclature as well as the reduced emphasis on multiplication tables—all changes that once had been acceptable under the aegis of academic mathematics, of mathematics for structure—were now dismissed as elitist frills. At times the criticism verged on the absurd. Claiming that the new math was a symptom of liberal excess simply because the program was embraced by John F. Kennedy and Lyndon B. Johnson is to fundamentally misunderstand the curriculum's origins as a deeply conservative, mathematician-driven response to claimed Cold War exigencies.

As with any curricular change, there is also the matter of "rollout," of the problems of teacher training and parental outreach, and of the greediness and sloppiness of professional textbook publishers. But such criticism alone does not explain the rapid change in opinion of the new math from 1958–1978. It was only after the politics changed that complaints about the curriculum took hold. The new math did not fail until it became the "wrong" way for kids to learn to reason.

That's why watching recent debates concerning the Common Core mathematics standards—or, for those with slightly longer memories, the "math wars" centered on the National Council of Mathematics Teachers' standards in the 1990s—can be so disheartening. Of course math, like all technical fields, requires both understanding and skillful symbolic manipulation. The evaluation of the curriculum in each case turned not on measures of how students learn best, or what specific mathematical topics are included, but

[11] Cambridge Conference on School Mathematics, *Goals for School Mathematics* (Boston, MA: Houghton Mifflin, 1963), 35.

[12] School Mathematics Study Group, *Mathematics for High School: Geometry, Teacher's Commentary*, rev. ed. (New Haven, CT: Yale University Press, 1961), 515–16.

[13] Jerome S. Bruner, *The Process of Education* (1960, repr., New York, NY: Vintage Books; 1963), 44.

[14] E. G. Begle, "The School Mathematics Study Group," *NASSP Bulletin* 43, no. 247 (1959), 26–31, on 27–28.

on the question of how students should learn to arrive at certain, incontrovertible knowledge. Critics of the new math wanted the mathematics of sets and structures to be replaced by rapid recall of arithmetic facts and memorization of multiplication tables. This was a statement both about the usefulness of mathematics and about its role in providing intellectual discipline. Lamoreaux did not deny that students would eventually get the right answer in her child's mathematics class; she wanted them to do it in the right way. She did not deny that there was virtue in creating standards; she wanted them to be created locally rather than imposed nationally. These are political issues, and therefore inherently contestable.

Rather than dismissing them as a dramatic aberration from the norms of math education, the Common Core math standards might be best understood as layered with sediment from previous math reform efforts. The instruction to emphasize "modeling with mathematics" or to encourage students to "look for and make use of structure" would be at home in, respectively, the 1923 Mathematical Association of America report and the School Mathematics Study Group's new math textbooks. The suggestion that "students should construct viable arguments and critique the reasoning of others" harkens back to George Birkhoff's 1930s textbooks and his claim that mathematical knowledge should enable students to make their reasoning more explicit. Encouragement to "make sense of problems and persevere in solving them" and to "use appropriate tools strategically" was nearly identical to George Pólya's postwar model of mathematics as a set of heuristics for problem solving.[15]

That previous conceptions of math's relevance and use are inscribed onto the standards is to be expected. But the rhetoric of the Common Core standards in 2009–2010 centered not on accumulated wisdom, but on advertising them as a new response to "years" of "stagnation." The standards promised to be uniquely and precisely formulated for the problems of the twenty-first century, making today's students "college- and career-ready." Mathematics' current usefulness is often subsumed within the larger claim that training in the so-called STEM fields—Science, Technology, Engineering, and Mathematics—is essential for getting a good job and ensuring a more prosperous nation. Though the overemphasis on these fields has come under increasing criticism as a way of further disadvantaging certain students or of keeping labor costs artificially low, the standards make a case for the crucial role mathematics can play in preparing a student for life after high school.[16] Like its predecessors, the Common Core recycled the ancient claim that mathematics is generally useful for intellectual discipline, even while suggesting that this way of teaching was precisely suited to the problems of contemporary society. Shackelford and Lamoreaux are entwined in a long history of complaints about the way math class teaches children to arrive at certain and reliable knowledge. But as long as learning math is presumed to discipline the mind generally, debates about math in classrooms will ultimately turn on the question of how students should learn to reason. And there is little hope that question will be resolved anytime soon.

[15] "Standards for Mathematical Practice," accessed April 27, 2016, http://www.corestandards.org/Math/Practice/; National Committee on Mathematical Requirements, *Reorganization of Mathematics* (ref. 6); George David Birkhoff and Ralph Beatley, *Geometry*, prelim. ed. (Boston, MA: Spaulding-Moss, 1933); G. Pólya, *How to Solve It: A New Aspect of Mathematical Method* (Princeton, NJ: Princeton University Press, 1961).

[16] For these criticisms specifically: Andrew Hacker *The Math Myth: And Other STEM Delusions* (New York, NY: The New Press, 2016); Michael S. Teitelbaum, *Falling Behind? Boom, Bust and the Global Race for Scientific Talent* (Princeton, NJ: Princeton University Press, 2014).